Alternating Cooperative Transmission for Energy-Efficient Broadcasting

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***Abstract*—We propose a broadcast protocol that is based on a form of cooperative diversity called the Opportunistic Large Array (OLA). In the initial broadcast, an SNR ("transmission") threshold is used to define mutually exclusive sets of OLAs, such that the union of the sets includes all the nodes in the network. The broadcast protocol then basically alternates the transmitting set of nodes (or OLAs) during each broadcast cycle and is called Alternating OLA with Transmission Threshold (A-OLA-T). Under A-OLA-T, broadcasting drains the energies of the nodes in the network efficiently and uniformly, extending the network life relative to broadcasts that use simple OLA or non-alternating OLAs with a transmission threshold. In this paper, we optimize the A-OLA-T protocol under the continuum assumption (very high node density).**

I. Introduction

BROADCASTING is a significant operation, especially in multi-hop networks [1]. *Energy-aware* broadcast protocols either minimize the energy consumption [3]-[5] or maximize network lifetime [6]-[7]. The objective of minimizing energy may not be the most efficient for a fixed source because if all the data packets are routed through the same minimum energy path, batteries along the path will drain quickly, while the remaining nodes in the network will remain intact. During broadcast, the network life can be maximized by routing the data packets such that the energy consumption is balanced among the nodes in the network. There are *cooperative* and *non-cooperative* algorithms that extend network life. Most broadcast algorithms in the current literature are non-cooperative and indirectly attempt to optimize groups of broadcasts to meet this objective. In this paper, we present broadcast strategies for multihop ad hoc and sensor networks that use cooperative transmission and explicitly optimize groups of broadcasts to increase the network longevity.

Cooperative transmission strategies provide spatial diversity, which enables dramatic reduction of the fade margins (i.e., the transmit powers) in a multipath fading environment, thereby saving energy [8], [9]. In [10], a simple cooperative transmission technique called the *Opportunistic Large Array* (OLA) was proposed, in which nodes behave without coordination between each other, but they naturally fire at approximately the same time in response to energy received from a single source or another OLA [11]. All the transmissions within an OLA are repeats of the same waveform; therefore the signal received from an OLA has the same model as a multipath channel. Small time offsets (because of different distances and computation times) and small frequency offsets (because each node has a different oscillator frequency) are like excess delays and Doppler shifts, respectively. As long as the receiver, such as a RAKE receiver, can tolerate the effective delay and Doppler spreads of the received signal and extract the diversity, decoding can proceed normally. Even though many nodes may participate in an OLA transmission, energy can still be saved because all nodes can reduce their transmit powers dramatically and large fade margins are not needed.

The authors gratefully acknowledge support for this research from the National Science Foundation under grant CNS-0721296.

OLA transmission has been proposed for energy-efficient broadcasting [10], [11], [13]-[15]. [10] and [11] propose what we refer to in this paper as Basic OLA. In Basic OLA, the first OLA comprises all nodes that can decode the transmission from the originating node; then the first OLA transmits and all nodes that can decode that transmission and that haven't decoded that message before, form the second OLA, and so forth.

The energy efficiency of OLAs can be improved by preventing those nodes from relaying, whose transmissions have a negligible effect on the formation of the next OLA. A node that receives much more power than is necessary for decoding is more likely to be near the source of the message. OLA with Transmission Threshold (OLA-T) method is simply Basic OLA with the additional transmission criterion that the node's received SNR must be *less* than a *user-specified* transmission threshold [18], [20], [22]. For a fixed source, such as the fusion node in a WSN, and for a static network, OLA-T causes the same subset of nodes to participate in all broadcasts. If we define network lifetime to be the length of time before the first node dies ("death" happens when the batteries die), and we assume that broadcasts are the only transmissions, then we observe that OLA-T has no advantage over Basic OLA in terms of network lifetime even though it consumes less total energy in a single broadcast.

The new broadcast scheme presented in this paper, which we call Alternating OLA With a Transmission Threshold (A-OLA-T), is an extension of OLA-T [18], [20], [22]. Unlike the OLA-based schemes above, our proposed strategy optimizes groups of broadcasts instead of a single broadcast. The key parameter is the *transmission* or SNR threshold, which controls the OLA sizes. The optimization involves minimizing the OLA sizes while utilizing mutually exclusive sets of OLAs on consecutive broadcasts, thereby balancing the broadcast load across the network. An important feature that A-OLA-T inherits from Basic OLA is that no individual nodes are addressed. This makes this protocol scalable with node density.

II. System Model

For our analysis, we adopt the notation and assumptions of [15]. Half-duplex nodes are assumed to be distributed uniformly and randomly over a continuous area with average node density ρ. The originating node is assumed to be a point source at the center of the given network area. We assume a node can *decode and forward* (DF) a message without error when its received signal-to-noise ratio (SNR) is greater than or equal to a modulation-dependent threshold [15]. Assumption of unit noise variance transforms the SNR threshold to a received power criterion, which is denoted as the decoding threshold τ_d. We note that the decoding threshold τ_d is not explicitly used in real receiver operations. A real receiver always just tries to decode a message. If no errors are detected, and the message was decoded properly, then it is assumed that the receiver power must have exceeded τ_d. In contrast, the Transmission Threshold that we will introduce later is used explicitly in the receiver to compare against the received SNR.

For simplicity, the *deterministic model* [15] is assumed, which means that the power received at a node is the sum of the powers from each of the node transmissions. This implies that signals received from different nodes are orthogonal. The orthogonality can be approximated for example, with Direct Sequence Spread Spectrum (DSSS) modulation, RAKE receivers, and by allowing transmitting nodes to delay their transmission by a random number of chips [16], [21]. Let the source power be denoted P_s, the relay transmit power be denoted P_r, and the relay transmit power per unit area be denoted by $\overline{P_r} = \rho P_r$. We assume a continuum of nodes in the network, which means that we let the node density ρ become very large ($\rho \to \infty$) while $\overline{P_r}$ is kept fixed. Continuing to follow [15], we assume a non-fading environment. The loss function in Cartesian coordinates is given by $l(x, y) = (x^2 + y^2)^{-1}$, where (x, y) are the normalized coordinates at the receiver. As in [15], distance d is normalized by a reference distance. Received power P_{rx}, from a node distance d away is $P_{\text{rx}} = \frac{P_0}{d^2}$[15], where P_0 is the power at $d = 1$. The aggregate path-loss from a circular disc of radius x at an arbitrary point q is given by $f(x, q) = \int_0^x \int_0^{2\pi} l(q - r\cos\theta, r\sin\theta) r dr d\theta$ [15]. The received power at a point q, P_q, is given by

$$P_q = \overline{P_r} \int_0^x \int_0^{2\pi} l(q - r\cos\theta, r\sin\theta) r dr d\theta. \quad (1)$$

The OLA-T method is simply Basic OLA with the additional criterion for relaying that the node's received SNR must be *less* than a specified transmission threshold, τ_b. The thresholds, τ_d and τ_b, define a range of received powers that correspond to the "significant" boundary nodes, which form the OLA. We define the Relative Transmission Threshold (RTT) as $\mathcal{R} = \frac{\tau_b}{\tau_d}$. Further, we define Decoding Ratio (DR) as $\mathcal{D} = \tau_d / \overline{P_r}$, because it can be shown to be the ratio of the receiver sensitivity (i.e. minimum power for decoding at a given data rate) to the power received from a single relay at the 'distance to the nearest neighbor,' $d_{nn} = 1/\sqrt{\rho}$. If ρ is a perfect square, then the d_{nn} would be the minimum distance between the nearest neighbors if the nodes were arranged in a uniform square grid.

We note that non-orthogonal transmissions in fading channels produce similarly shaped OLAs [15], therefore the A-OLA-T concept should work for them as well, although the theoretical results would have to be modified.

III. Alternating OLA-T (A-OLA-T)

In this Section, we propose the Alternating OLA-T (A-OLA-T), which improves the network lifetime compared to Basic OLA and OLA-T. In A-OLA-T, broadcasts are grouped. Any number of broadcasts may be grouped under the continuum assumption; with finite node density, smaller group sizes are expected to be the best to ensure that the OLAs are populated with a sufficient number of nodes. In this paper, we consider just two groups, called *Broadcast 1* and *Broadcast 2*.

The idea of A-OLA-T is that the nodes that do not participate in one broadcast make up the OLAs in the next broadcast. To ensure that the sets of OLAs during each broadcast are mutually exclusive, the OLA boundaries should not change during the two broadcasts. It remains to determine if there exist OLA radii for A-OLA-T such that both Broadcasts 1 and 2 are successful, where success means that the broadcasted message propagates to the edge of the network. Since the boundaries don't change, our approach will be to first review the sufficient condition for Broadcast 1 to be successful. This condition takes the form of a lower bound on $\mathcal{R}$. A $\mathcal{R}$ that satisfies this bound fixes the boundaries. Next, we derive a necessary and sufficient condition for Broadcast 2 to also be successful given these boundaries. The second condition is an upper bound on $\mathcal{R}$.

A. Broadcast 1 (OLA-T) [18]

Broadcast 1 is just OLA-T from previous work [18], which is summarized as follows.

Let the radii sequences $\{r_{d,k}\}$ and $\{r_{b,k}\}$ denote the outer and inner boundary radii sequences, respectively, for the k-th OLA ring formed during the Broadcast 1, as shown in the upper half of Fig. 1(a) (OLAs are indicated with blue shading). The boundaries can be found recursively using

$$\overline{P_r}\left[f(r_{d,k}, r_{j,k+1}) - f(r_{b,k}, r_{j,k+1})\right] = \tau_j, \ j \in \{b, d\}. \quad (2)$$

Using the initial conditions, $r_{d,1} = \sqrt{\frac{P_s}{\tau_d}}$ and $r_{b,1} = \sqrt{\frac{P_s}{\tau_b}}$, the definitions for the k-th OLA using a recursive formula are given by

$$r_{d,k}^2 = \frac{\beta(\tau_d) r_{d,k-1}^2 - r_{b,k-1}^2}{\beta(\tau_d) - 1}, \ r_{b,k}^2 = \frac{\beta(\tau_b) r_{d,k-1}^2 - r_{b,k-1}^2}{\beta(\tau_b) - 1}. \quad (3)$$

From [18], the closed-form expressions for OLA-T radii are given by

$$r_{d,k}^2 = \frac{\eta_1 A_1^{k-1} - \eta_2 A_2^{k-1}}{A_1 - A_2}, \ r_{b,k}^2 = \frac{\zeta_1 A_1^{k-1} - \zeta_2 A_2^{k-1}}{A_1 - A_2}, \quad (4)$$

where

$$A_1 = \alpha(\tau_d) - \alpha(\tau_b), \quad A_2 = 1, \tag{5}$$

$$\eta_i = \left\{ [A_i + \alpha(\tau_b)] \frac{P_s}{\tau_d} - \alpha(\tau_d) \frac{P_s}{\tau_b} \right\},$$

$$\zeta_i = \left\{ [1 + \alpha(\tau_b)] \frac{P_s}{\tau_d} + [A_i - \alpha(\tau_d) - 1] \frac{P_s}{\tau_b} \right\},$$

$$\alpha(\tau) = [\beta(\tau) - 1]^{-1}, \; \beta(\tau) = \exp\left[\tau/(\pi \overline{P_r})\right],$$

$$i \in \{1, 2\}, \text{ and } A_1 - A_2 \neq 0.$$

From [22], it is learned that a necessary and sufficient condition to achieve infinite network broadcast with a constant transmission threshold is the inequality,

$$2 \quad \geq \quad \exp\left(\frac{\mathcal{D}}{\pi}\right) + \exp\left(\frac{-\mathcal{D}\mathcal{R}}{\pi}\right), \tag{6}$$

which takes the form of the following lower bound for $\mathcal{R}$

$$\mathcal{R}_{\text{lower bound}} = (-1) \left\{ \frac{\pi \ln\left[2 - \exp\left(\frac{\mathcal{D}}{\pi}\right)\right]}{\mathcal{D}} \right\}. \tag{7}$$

We observe that when $\mathcal{R} \to \infty$, OLA-T becomes Basic OLA, and (6) becomes

$$2 \geq \exp\left(\frac{\mathcal{D}}{\pi}\right), \tag{8}$$

which is the condition for successful Basic OLA broadcast [15].

B. Necessary and Sufficient Condition for Broadcast 2 Success

During Broadcast 2, the set of nodes that transmitted during Broadcast 1 will not transmit and the nodes that did not participate during the the first broadcast will transmit. In the previous Section, we presented a lower bound on $\mathcal{R}$. In this Section, we show that an upper bound on $\mathcal{R}$ is required for a successful Broadcast 2.

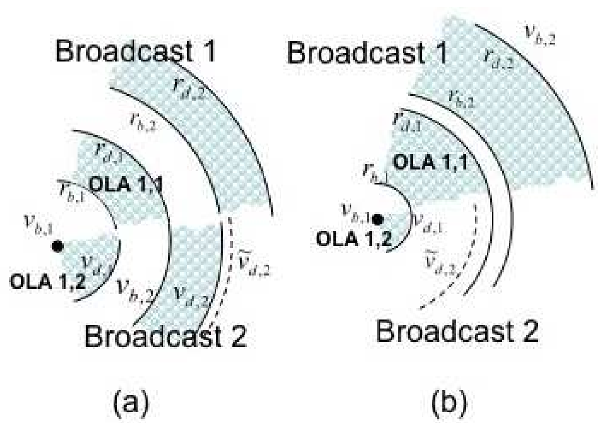

Fig. 1. *Illustration of the A-OLA-T Algorithm with (a) admissible $\mathcal{R}$, (b) inadmissible $\mathcal{R}$.*

Fig. 1(a) and (b) illustrate how it is possible to design OLAs for Broadcasts 1 and 2, to ensure that their propagation is sustained. The upper part of both drawings corresponds to Broadcast 1 and the radii are labeled according to (3). The lower parts of both drawings correspond to Broadcast 2. In Fig. 1(a) the OLA radii are relabeled $\{v_{d,k}\}$ and $\{v_{b,k}\}$, to denote the outer and inner boundary radii sequences, respectively for the k-th OLA ring formed during the Broadcast 2. The initial conditions for the second broadcast are $v_{b,1} = 0$, and $v_{d,1} = \sqrt{\frac{P_s}{\tau_b}}$. In Fig. 1(a), the first OLA during Broadcast 1 is denoted by *OLA 1,1* and is defined by the radii pair, $\{r_{b,1}\}$ and $\{r_{d,1}\}$. On the other hand, the first OLA during Broadcast 2 is denoted by *OLA 1,2* and is the circular disk of radius $\{v_{d,1}\}$. Let $\widetilde{v}_{d,2}$ be the decoding range of *OLA 1,2* during Broadcast 2. The key idea is that $\widetilde{v}_{d,2}$ must be greater than $r_{b,2}$. In Fig. 1(a), this inequality is satisfied, while in Fig. 1(b), it is not. More generally, the network designer just needs to check that the decoding range, $\widetilde{v}_{d,k+1}$, of the k-th OLA in Broadcast 2 is always greater than $r_{b,k+1}$, for all k. Alternatively, we can compute the received power at $r_{b,k+1}$ and confirm that it is greater than the minimum. Mathematically, we express this as

$$\overline{P_r} \left[f(v_{d,k}, r_{b,k+1}) - f(v_{b,k}, r_{b,k+1}) \right] \geq \tau_d. \tag{9}$$

We then substitute $r_{b,k} = v_{d,k}$ and $r_{d,k-1} = v_{b,k}$.

Intuitively, we observe that as $\mathcal{R}$ becomes very large, the OLAs during Broadcast 1 become larger and the OLAs of Broadcast 2 become relatively smaller, as shown in Fig. 1(b). As a result, the sets of nodes that did not transmit during Broadcast 1 (or the OLAs during Broadcast 2), eventually become so small that their decoding range (indicated by the dashed line in Fig. 1(b)) cannot reach the next Broadcast 2 OLA to sustain propagation. In other words, for a very high value of $\mathcal{R}$, the k-th OLA in Broadcast 2 may be so weak that some nodes between $v_{b,2}$ and $v_{d,2}$ cannot decode the signal. When this happens, OLA formations die off during Broadcast 2 and A-OLA-T fails to achieve network broadcast. Thus, it makes sense for $\mathcal{R}$ to have an upper bound. In the remainder of this section, we provide highlights of a derivation that guarantees a successful Broadcast 2. The complete derivation is in [23].

Substituting $r_{b,k} = v_{d,k}$ and $r_{d,k-1} = v_{b,k}$ into (9), and using the same approach as in [18] we can rewrite (9) as follows.

$$\begin{aligned} r_{b,k+1}^2 \quad &\leq \quad \frac{\beta(\tau_d) v_{d,k}^2 - v_{b,k}^2}{\beta(\tau_d) - 1} = \frac{\beta(\tau_d) r_{b,k}^2 - r_{d,k-1}^2}{\beta(\tau_d) - 1}, \\ \Rightarrow 0 \quad &\leq \quad \frac{\beta(\tau_d) r_{b,k}^2 - r_{d,k-1}^2 - (\beta(\tau_d) - 1)\, r_{b,k+1}^2}{\beta(\tau_d) - 1}. \end{aligned}$$

Next, we substitute the expressions for $r_{d,k}$ and $r_{b,k}$ from (4). Using the relation (6), collecting the A_1 and A_2 terms and re-arranging, we get

$$0 \leq \Big\{ A_1^{k-1} \Big[\Big(\alpha(\tau_d) + 1 \Big) \zeta_1 - \alpha(\tau_d) \eta_1 A_1^{-1} - \zeta_1 A_1 \Big] - A_2^{k-1} \Big[\Big(\alpha(\tau_d) + 1 \Big) \zeta_2 - \alpha(\tau_d) \eta_2 A_2^{-1} - \zeta_2 A_2 \Big] \Big\}.$$

Next, we re-write this as shown below.

$$A_1^{k-1} \Omega - A_2^{k-1} \Pi \geq 0. \tag{10}$$

where

$$\Omega = \Big(\alpha(\tau_d) + 1 \Big) \zeta_1 - \alpha(\tau_d) \eta_1 A_1^{-1} - \zeta_1 A_1, \text{ and}$$

$$\Pi = \Big(\alpha(\tau_d) + 1 \Big) \zeta_2 - \alpha(\tau_d) \eta_2 A_2^{-1} - \zeta_2 A_2.$$

Using $A_2 = 1$ from (5), we get $\Pi = \zeta_2 - \eta_2 = 0$, which, when applied to (10) along with $A_1 > 1$ (proved in [23]), (10) may be simplified to $\Omega \geq 0$.

The inequality in (10) implies an upper bound on $\mathcal{R}$, the closed-form expression for which has been derived in [23] and is given by

$$\mathcal{R}_{\text{upper bound}} = \frac{\pi \ln(r_1)}{\mathcal{D}}, \tag{11}$$

where

$$r_1 = \frac{\beta(\tau_d) + 1 + \sqrt{\Big(\beta(\tau_d) + 1 \Big)^2 - 4}}{2}.$$

We remark that it is not necessary to assume the same constant $\mathcal{R}$ for both broadcasts or even for a single broadcast [20]. With the flexibility of variable transmission thresholds (τ_b^k or $\mathcal{R}_k$), a designer may be able to make the decoding ranges in Broadcast 2 match up with the boundaries in Broadcast 1.

C. Relationship Between the Bounds and Relay Power

Fig. 2 is a plot of the upper and lower bounds for relative transmission threshold, $\mathcal{R}$, in dB for A-OLA-T, as a function of the decoding ratio, $\mathcal{D}$. First, we observe that as $\mathcal{D}$ decreases, the difference between the upper and lower bounds increases. As an example, for a small decrease in $\mathcal{D}$ from 1.2 to 1, the range of $\mathcal{R}$ increases from $[2.1, 2.4]$ to $[1.7, 2.8]$. This has two reasons. Decreasing $\mathcal{D}$ could be done by increasing the $\overline{P_r}$, which enables Broadcast 1 to be successful with more slender OLAs. This corresponds to a decrease of the lower bound. Fatter Broadcast 2 OLAs more easily reach across the next pair of boundaries and so this increases the upper bound. Next, decreasing τ_d also decreases $\mathcal{D}$. Decreasing τ_d decreases the lower bounds, because a lower value of τ_d corresponds to a lower SNR requirement at the receiving node, and so in order to meet this power requirement, the OLAs must include more nodes during Broadcast 1. This is achieved by increasing $\mathcal{R}$. So, OLAs during Broadcast 1 become thinner but more powerful, and the OLAs during Broadcast 2 grow thicker.

We also observe from Fig. 2 that the upper and lower bounds converge as $\mathcal{D}$ increases. This also implies an upper bound on $\mathcal{D}$ for A-OLA-T, $\mathcal{D}_{\max}^{(\text{A})} = \frac{\tau_d}{\overline{P_r}_{\min}^{(\text{A})}}$, where $\overline{P_r}_{\min}^{(\text{A})}$ is the minimum

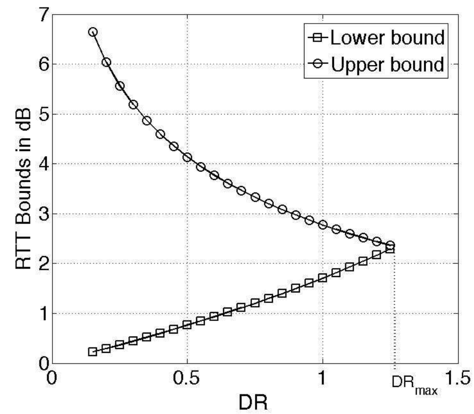

Fig. 2. *RTT, $\mathcal{R}$, in dB, Versus DR for A-OLA-T, $\mathcal{D}$. The $\mathcal{D}$ corresponding to the intersection of the two curves is the $\mathcal{D}_{max}^{(A)}$.*

value of $\overline{P_r}$ for a given τ_d. We were not able to obtain an exact value of $\mathcal{D}_{\max}^{(\text{A})}$, however, using numerical analysis we found $\mathcal{D}_{\max}^{(\text{A})} \approx 1.27$. We note from (8) that $\mathcal{D}$ has a higher upper bound for Basic OLA, $\mathcal{D}_{\max}^{(\text{O})} = \pi \ln(2) \approx 2.18$. For $\mathcal{D} > \mathcal{D}_{\max}^{(\text{A})}$, network broadcast fails for A-OLA-T because the OLAs die out during Broadcast 2. For A-OLA-T, we have from $\mathcal{D}_{\max}^{(\text{A})}$ that $\overline{P_r}_{\min}^{(\text{A})} \simeq 0.78\tau_d$. For Basic OLA, the minimum $\overline{P_r}$, denoted by $\overline{P_r}_{\min}^{(\text{O})}$ as a function of τ_d can be found directly from (8): $\overline{P_r}_{\min}^{(\text{O})} = 0.46\tau_d$. We observe that A-OLA-T requires slightly less than double the power of Basic OLA, because it is using fewer nodes.

Next, we compute the "broadcast life" extension of A-OLA-T compared to Basic OLA. By broadcast life, we mean the lifetime of the network if only broadcasts were transmitted. At a first glance, it might seem that A-OLA-T doubles the battery life of the sensors in the network compared to Basic OLA. This is true if A-OLA-T and Basic OLA use the same $\overline{P_r}$. However, this would not be a fair comparison since Basic OLA can achieve successful broadcast at a lower $\overline{P_r}$. Since for a given protocol, all nodes use the same amount of power in broadcasts, we assume the broadcast life of the network is inversely proportional to the time-averaged power transmitted by each node. For Basic OLA, the time-averaged power is $\overline{P_r}^{(\text{O})}$. For A-OLA-T with two sets, the time-averaged power is $\frac{\overline{P_r}^{(\text{A})}}{2}$, since each node transmits only every other broadcast. The ratio of broadcast lives of Basic OLA to A-OLA-T is therefore $2\frac{\overline{P_r}^{(\text{O})}}{\overline{P_r}^{(\text{A})}}$, and the 'Fraction of Life Extension' (FLE), may be defined as

$$\text{FLE} = 2\frac{\overline{P_r}^{(\text{O})}}{\overline{P_r}^{(\text{A})}} - 1. \tag{12}$$

FLE can be evaluated for any powers that satisfy $\overline{P_r}^{(\text{A})} \geq 0.78\tau_d$ and $\overline{P_r}^{(\text{O})} \geq 0.46\tau_d$. However, when the the minimum powers are substituted, then (12) becomes

$$\widehat{\text{FLE}} = 2\frac{\overline{P_r}^{(\text{O})}_{\min}}{\overline{P_r}^{(\text{A})}_{\min}} - 1 = 2\frac{\mathcal{D}^{(\text{A})}_{\max}}{\mathcal{D}^{(\text{O})}_{\max}} - 1 \approx 0.17, \qquad (13)$$

where $\widehat{\text{FLE}}$ represents the FLE achieved by A-OLA-T relative to Basic OLA when both protocols operate in their minimum power configurations. This means that A-OLA-T can offer a 17% life extension when both protocols are optimized.

Finally, we show that the FLE result is consistent with the 'Fraction of Energy Saved' (FES) computed for OLA-T in [22]. The definition for the FES [1] of OLA-T relative to Basic OLA [22], is

$$\begin{aligned} \text{FES} &= 1 - \frac{\Big(\text{Energy consumed in one broadcast}\Big)_{\text{OLA-T}}}{\Big(\text{Energy consumed in one broadcast}\Big)_{\text{Basic OLA}}}, \\ &= 1 - \frac{\overline{P_r}^{(\text{A})} \sum_{k=1}^{L} \left(r^2_{d,k} - r^2_{b,k}\right)}{\overline{P_r}^{(\text{O})} r^2_{d,L}}, \end{aligned} \qquad (14)$$

where L is the number of OLAs in the OLA-T network.

If OLA-T operates in the minimum power configuration for two-set A-OLA-T Broadcast 1, and so uses the $\overline{P_r}^{(\text{A})} = \overline{P_r}^{(\text{A})}_{\min}$, then we know from Appendix B of [23] that the ratio of areas in (14) is approximately $1/2$, and we have

$$\text{FES} = 1 - \frac{\overline{P_r}^{(\text{A})}_{\min}}{\overline{P_r}^{(\text{O})}}\left(\frac{1}{2}\right) = 1 - \frac{\mathcal{D}^{(\text{O})}}{\mathcal{D}^{(\text{A})}_{\max}}\left(\frac{1}{2}\right). \qquad (15)$$

So, when Basic OLA is optimized, i.e. when $\mathcal{D}^{(\text{O})} = \mathcal{D}^{(\text{O})}_{\max}$, (15) can be related to (13)

$$\text{FES}_{\text{OLA-T}} = \frac{\widehat{\text{FLE}}}{\widehat{\text{FLE}} + 1} \approx \frac{0.17}{0.17 + 1} = 0.145, \qquad (16)$$

where $\text{FES}_{\text{OLA-T}}$ describes the FES achieved by OLA-T relative to Basic OLA. The FES that is computed using (16) agrees with the value that can be read off the "500 levels" curve of Fig. 4 of [22] for Decoding Ratio (DR) $= \mathcal{D}^{(A)}_{\max} = 1.27$.

IV. Conclusions

In this paper, we proposed and analyzed a novel same-source broadcast strategy that extends the life of a wireless ad hoc or sensor network by alternating between two mutually exclusive sets of opportunistic large arrays (OLAs) in pairs of broadcasts. We showed that A-OLA-T extends the network life by a maximum of 17% relative to the Basic OLA. Further, when A-OLA-T is compared to OLA-T, the battery-life of the nodes is doubled. The key parameter is the transmission threshold, which was assumed constant for the whole network. Plans for future work include an analysis of A-OLA-T for finite densities of nodes, other path-loss exponents, and fading environments, and a consideration of the limitations of practical synchronization.

[1] We note that definition of FES in this paper and in [22] is different from the definition in [18]-[20]. In [18]-[20], we assume $\overline{P_r}$ for OLA-T and Basic OLA to be the same.

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